Tests of Conditional Predictive Ability: Some Simulation Evidence

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This paper ...

[1] Provide data-generating-processes (DGPs) that are consistent with Giacomini and White (2006, *Econometrica*)'s theoretical framework

[2] Analyze finite-sample properties of Giacomini and White (GW, hereafter)'s testing procedures for

1. Unconditional predictive ability
2. Conditional predictive ability

where forecasts are generated with

1. Fixed-window estimation scheme
2. Rolling-window estimation scheme

In this discussion

I will talk about this paper in terms of ...

1. Relevance
2. Challenges

Then, I will list my comments on the paper

Relevance

Among 50 regular articles published over the last year in the *International Journal of Forecasting*:

- 25 papers perform statistical testing on equal-predictive-ability
- 20 papers report results based on Diebold-Mariano (DM)-type statistic with
  - Recursive-window estimation (11 papers)
  - **Rolling-window estimation (8 papers)**
  - **Fixed-window estimation (1 paper)**

Challenges

Forecast target: y_{t+h}

Two forecasts made at time t: $f_{1,t}$ and $f_{2,t}$

Define a loss differential

$$d_{t+h} = L(y_{t+h}, f_{1,t}) - L(y_{t+h}, f_{2,t})$$

where $L(y, f)$ is a loss function

Consider two types of null hypotheses for equal predictive ability

$$E[d_{t+h}] = 0 \quad \text{or} \quad E[d_{t+h}|\mathcal{F}_t] = 0$$

Challenges

Conditional predictive ability test:

$$E[d_{t+h}|\mathcal{F}_t] = 0$$

GW's theory is based on a specific form of $\mathcal{F}_t$

They assume that $f_{1,t}$ and $f_{2,t}$ are measurable with respect to $\mathcal{F}_t$

For 1-step-ahead prediction,

- (d_{t+1}) becomes a martingale difference sequence (MDS)
- $(d_{t+1} z_t)$ becomes a MDS where $z_t \in \mathcal{F}_t$
- Their asymptotic theory boils down to LLN/CLT for MDS

For h-step-ahead prediction, d_{t+h} is serially correlated only up to $(h-1)$ displacements

Challenges

Is this assumption realistic?

Assume a quadratic loss function,

$$d_{t+1} = (y_{t+1} - f_{1,t})^2 - (y_{t+1} - f_{2,t})^2$$

Simple arrangement shows that GW's null hypothesis holds when

$$E[y_{t+1}|\mathcal{F}_t] = \frac{1}{2}(f_{1,t} + f_{2,t})$$

For example, if $f_{1,t}$ is unbiased, then $f_{2,t}$ has to be unbiased.

It is hard to come up with such an example at least for me. But, the author came up with such an example

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Victory!

Comment 1: HAC-Bandwidth

Comment 1: If the paper was really about documenting a finite sample properties of GW's test procedures, then it is much more natural to **use a rectangular kernel rather than a Bartlett kernel** to estimate the long-run variance for "conditional" EPA test

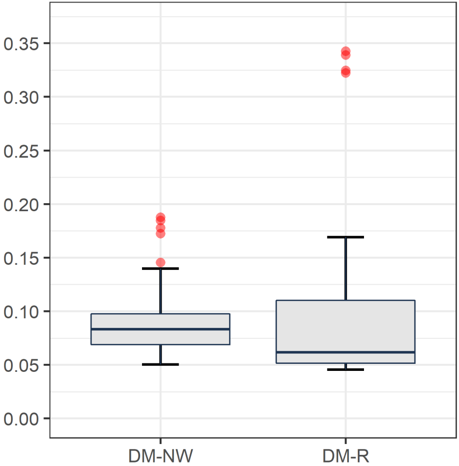
- Giacomini and White (2006) emphasized this in the paper. "Use HAC and truncate at $(h-1)$ for the conditional testing."
- Also emphasized in Giacomini (2010)

Under the conditional null hypothesis, it is better because ...

- Smaller size distortion due to less biased LRV estimation
- More powerful if the alternative entails a serial correlation

Comment 1: HAC-Bandwidth

Empirical distribution of actual sizes for all experiments considered in the author's paper (all 80 specifications), **nominal size = 5%**



- **DM-NW (This paper)**: Bartlett kernel with Newey and West's automatic bandwidth rule
- **DM-R (GW's original rule)** : Rectangular kernel truncated at $(h - 1)$

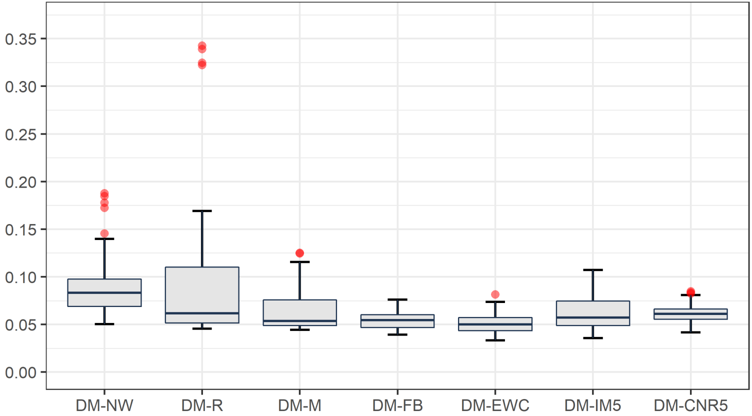
Comment 2: On size distortion

Comment 2: Most of size distortions that we saw from the author's exercises can be eliminated by using different approaches

- Recent tools from HAR-inference literature: Fixed-b asymptotics, t-statistic-based testing, randomization test using asymptotic symmetry, etc

- Accurate estimation of the long-run variance may not be an issue here (e.g., fixed-b asymptotics)

Comment 2: On size distortion



- DM-NW: This paper
- DM-R: GW's original rule
- DM-M: Harvey et al. (1997)
- DM-FB: Kiefer and Vogelsang (2005)
- DM-EWC: Muller (2004)
- DM-IM: Ibragimov and Muller (2010)
- DM-CNR: Canay, Romano, Shaikh (2017)

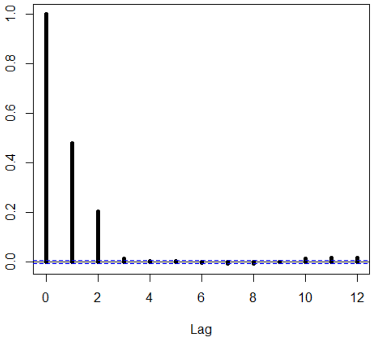
Comment 3: Interesting property of the DGP

Comment 3: Conditional-Rolling DGP generates a very strong conditional heteroscedasticity in the loss differential

Comment 3: Interesting property of the DGP

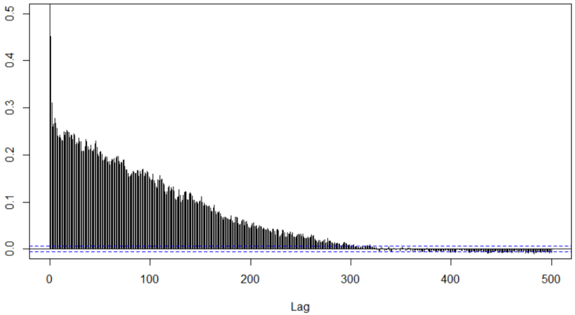
For example, R = 175; Rbar = 175; h = 3; with P = 1000000;

ACF of d_{t+h} is MA(2) as expected



Comment 3: Interesting property of the DGP

ACF of d_{t+h}^2



Fitting MA(2)-GARCH(1,1) gives me the following estimates

$$\omega = 0.00, \quad \beta_1 = 0.23, \quad \beta_2 = 0.81$$

where the GARCH equation is defined as

$$\sigma_t^2 = \omega + \beta_1 e_{t-1}^2 + \beta_2 \sigma_{t-1}^2$$

Comment 4: My personal view about GW

Conditional EPA testing:

$$E[d_{t+h}|\mathcal{F}_t] = 0$$

GW's theory is based on a specific form of $\mathcal{F}_t$

- For example, $f_{1,t}$ and $f_{2,t}$ are measurable with respect to $\mathcal{F}_t$
- Theoretically convenient, more powerful test, etc.
- It makes sense if you want to know whether d_{t+h} is predictable
- However, for this null to hold we have to impose a very strong restrictions on $f_{1,t}$ and $f_{2,t}$

Comment 4: My personal view about GW

GW's conditional testing also used for explanatory analysis:

- Is d_{t+1} statistically different from zero during recession?
- Is d_{t+1} is associated with some economic variables (z_t)?

One could perform a test based on

$$d_{t+h} = \beta_0 + \beta_1 z_t + \varepsilon_{t+h}$$

$E[\varepsilon_{t+h}] = 0$ and $E[\varepsilon_{t+h} \times z_t] = 0$

Then, just test $\beta_0 = 0$ and $\beta_1 = 0$ with HAR inference **WITHOUT “truncation at (h-1)”** admitting that ε_{t+h} can exhibit higher-order serial correlation

Conclusion

- This paper is very relevant for forecasters
- This paper attacks a challenging problem
- This paper offers several useful lessons for users of GW's tests