Bayesian time-varying VARs for forecasting and structural analysis

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[1] **Disclaimer:** The views expressed here are my own and do not necessarily represent the views of the Federal Reserve Bank of Philadelphia or the Federal Reserve System.

This talk is based on two papers

1. Macroeconomic Forecasting and Variable Ordering in Multivariate Stochastic Volatility Models (joint with Jonas Arias and Juan Rubio-Ramirez), 2023, *Journal of Econometrics*.

2. Inference Based on Time-Varying SVARs Identified with Sign Restrictions (joint with Jonas Arias, Juan Rubio-Ramirez, and Dan Waggoner), 2024, Working Paper.

How to place a prior on unknowns in time-varying VARs?

A standard reduced-form vector autoregressions (VARs)

A reduced-form VAR model with n variables and p lags:

$$y_t' = x_t' B + e_t', \quad e_t \sim N(0, \Sigma).$$

- y_t and e_t are $n \times 1$ vectors
- $x_t = [1_{n\times 1}, y_{t-1}, y_{t-2}, ..., y_{t-p}]'$, a $1 \times (1 + np)$ vector
- B is a $n \times (1 + np)$ matrix
- Σ is a $n \times n$ matrix

Prior:

$$B \sim N(m_B, V_B)$$
$$\Sigma \sim IW(S_\Sigma, v_\Sigma)$$

Time-varying reduced-form VARs

A time-varying reduced-form VAR model,

$$y_t' = x_t' B_t + e_t', \quad e_t \sim N(0, \Sigma_t)$$

It is useful for both forecasting and structural analysis:

- Forecasting: West and Harrison (1997), Clark (2011), D'Agostino, Gambetti, Giannone (2013), Koop and Korobilis (2013), ... and many others.
- Structural analysis: Primiceri (2005), Sims and Zha (2006), Baumeister and Peersman (2013), Bognanni (2018), ... and many others.

Placing a prior on time-varying unknown objects like $B_{1:T}$ and $\Sigma_{1:T}$ is challenging.

Roadmap for the rest of today's talk

1. Begin with a standard prior specification for time-varying VARs.
2. Deviate from it and consider other alternatives (**reduced-form VAR**)
3. Extend these priors for a more general structural analysis (**structural VAR**)
4. Application: The recent monetary policy tightening cycle.

[1] Priors for time-varying reduced-form VARs

Macroeconomic Forecasting and Variable Ordering in Multivariate Stochastic Volatility Models (joint with Jonas Arias and Juan Rubio-Ramirez), 2023, *Journal of Econometrics*.

A popular Bayesian approach for time-varying VARs

Model: A time-varying reduced-form VAR

$$y_t' = x_t' B_t + e_t', \quad e_t \sim N(0, \Sigma_t), \quad \text{for } t = 1, ..., T.$$

Prior: Primiceri (2005) develops a prior specification that models time-varying parameters as a function of time, and imposes a Gaussian process prior,

$$vec(B_t) = vec(B_{t-1}) + \nu_t, \quad \nu_t \sim N(0, V_B)$$

and

$$\Sigma_t = L_t \Omega_t L_t'$$

where L_t is lower triangular matrix (with ones on the diagonal) and Ω_t is a diagonal matrix with

$$vecl(L_t) = vecl(L_{t-1}) + \zeta_t, \quad \zeta_t \sim N(0, V_C)$$
$$log(diag(\Omega_t)) = log(diag(\Omega_{t-1})) + \eta_t, \quad \eta_t \sim N(0, V_\Omega)$$

A simple example, 1

Consider a simple example with two variables—real GDP growth (Δy_t) and the federal funds rate (r_t)—and without lags or constant ($x_t = 0$):

Model:

$$\Delta y_t = e_t^y$$
$$r_t = e_t^r$$

where $[e_t^y, e_t^r]' \sim N(0, \Sigma_t)$

Prior:

$$\Sigma_t = \begin{pmatrix} 1 & 0 \\ \ell_t & 1 \end{pmatrix} \begin{pmatrix} \sigma_{y,t}^2 & 0 \\ 0 & \sigma_{r,t}^2 \end{pmatrix} \begin{pmatrix} 1 & \ell_t \\ 0 & 1 \end{pmatrix}$$
$$\log(\sigma_{y,t}^2) = \log(\sigma_{y,t-1}^2) + \eta_{y,t}$$
$$\log(\sigma_{r,t}^2) = \log(\sigma_{r,t-1}^2) + \eta_{r,t}$$
$$\ell_t = \ell_{t-1} + \zeta_t$$

Cholesky factorization makes imposing a Gaussian process type prior easier, but ...

A simple example, 2

Cholesky factorization leads to a recursive structure

$$\begin{aligned}\Delta y_t &= \sigma_{y,t}\varepsilon_t^y \\ r_t &= \ell_t\sigma_{y,t}\varepsilon_t^y + \sigma_{r,t}\varepsilon_t^r\end{aligned}$$

where ε_t^y and ε_t^r are independent standard normal random variables.

Conditional predictive distribution conditional on $\sigma_{y,t}, \sigma_{r,t}, \ell_t$ is

- Normal distribution for Δy_t.
- Mixture of normal distributions for r_t.

It introduces an asymmetry in the distributional assumptions.

Does ordering matter in practice?

This is acknowledged by many others including Primiceri (2005).

However, it was less known how relevant it is in practice.

- Pseudo real-time out-of-sample forecasting evaluation
- 4-variable VAR with 2 lags in quarterly frequency,
  - output growth (real GDP growth), inflation (core PCE inflation), 3-Month T-Bill, unemployment rate
- Recursively estimate TV reduced-form VAR with Primiceri (2005)'s prior and generate forecasts:
  - evaluation sample runs from 1987Q2 to 2018Q4 (120 quarters)
- We do this for all 24 orderings and ranking them by various forecasting evaluation metrics

Density prediction evaluation

Log Predictive Score, One-Quarter-Ahead

	Min	Max	Median
Output Growth	-281.64	-274.42	-278.47
Inflation	-113.90	-111.27	-111.96
3-Month T-Bill	-28.81	-10.34	-14.83
Unemployment	21.12	30.25	27.09
Joint	-381.08	-350.88	-359.67

- Min, Max, Median LPSs based on 24 orderings.
- The difference in terms of density prediction can be substantial.

Alternative priors that are ordering invariant

Two classes of alternative priors that are "ordering invariant"

1. Model Σ_t based on Wishart(-like) distribution.
   - A probabilistic model for transition from Σ_{t-1} to Σ_t.
   - For examples, West and Harrison (1997), Uhlig (1997), Prado and West (2010), Wu and Koop (2023).

2. Factorize Σ_t in a different way. Then, use a similar Gaussian process prior.
   - $\Sigma_t = D_t C_t D_t$ where D_t is a diagonal matrix and C_t is a correlation matrix.
   - For example, Engle (2002), Asai and McAleer (2009).

Random correlations VAR

Arias, Rubio-Ramirez, and Shin (2023) introduces a new class of models, random correlations VAR (RC-VAR),

$$y_t = B_t x_{t-1} + e_t, \quad e_t \sim N(0, \Sigma_t)$$

Random correlations VAR:

$$\Sigma_t = D_t C_t D_t$$
$$\delta_t = 2\log(diag(D_t))$$
$$\gamma_t = \mathcal{G}(C_t)$$

Prior:

$$vec(B_t) = vec(B_{t-1}) + \nu_t, \quad \nu_t \sim N(0, V_B)$$
$$\delta_t = \delta_{t-1} + \eta_t, \quad \eta_t \sim N(0, V_\delta)$$
$$\gamma_t = \gamma_{t-1} + \zeta_t, \quad \zeta_t \sim N(0, V_\gamma)$$

The mapping from C_t to γ_t is studied by Archakov and Hansen (2021).

From correlation matrix to real vector

$$\gamma = vecl(log(C)) \in \mathbb{R}^{n(n-1)/2}$$

where

- $vecl()$ is the vectorization operator of the lower off-diagonal elements of the input matrix.
- $log(C) = V \log(\Lambda) V'$ where $C = V \Lambda V'$.

The mapping from C to γ is studied by Archakov and Hansen (2021).

- Each element in γ lies on the real line.
- Its inverse mapping exists.

Density prediction evaluation, revisited

Log Predictive Score, One-quarter-ahead

	Primiceri (2005)'s TV-VAR			
	Min	Max	Median	RC-VAR
Joint	-381.08	-350.88	-359.67	-362.05
Output Growth	-281.64	-274.42	-278.47	-279.30
Inflation	-113.90	-111.27	-111.96	-112.83
3-Month T-Bill	-28.81	-10.34	-14.83	-12.20
Unemployment	21.12	30.25	27.09	25.41

- RC-VAR performs on par with the Median model.

[2] Priors for time-varying structural VARs

Inference Based on Time-Varying SVARs Identified with Sign Restrictions (joint with Jonas Arias, Juan Rubio-Ramirez, and Dan Waggoner), 2024, Working Paper.

A class of models for structural analysis

A "structural" VAR model,

$$y_t' A = x_t' F + \varepsilon_t', \quad \varepsilon_t \sim N(0, I)$$

where

- A and F are "structural" parameters
- ε_t is a vector of "structural" shocks

A simple example: a structural VAR without time-variation

To fix ideas, consider a simple example with two variables—the federal funds rate (r_t) and real GDP growth (Δy_t)—and without lags or constant ($x_t = 0$):

$$\begin{aligned} r_t &= \psi \Delta y_t + \sigma^{MP} \varepsilon_t^{MP} \\ \Delta y_t &= -\alpha r_t + \sigma^{D} \varepsilon_t^{D} \end{aligned}$$

This can be written as,

$$\underbrace{[r_t,\ \Delta y_t]}_{y_t'} \underbrace{\begin{pmatrix} \frac{1}{\sigma^{MP}} & \frac{\alpha}{\sigma^{D}} \\ -\frac{\psi}{\sigma^{MP}} & \frac{1}{\sigma^{D}} \end{pmatrix}}_{A} = \underbrace{[\varepsilon_t^{MP},\ \varepsilon_t^{D}]}_{\varepsilon_t'}$$

A simple example with time-variation

The same model but with time-varying parameters

$$
\begin{aligned}
r_t &= \psi_t \Delta y_t + \sigma_t^{MP} \varepsilon_t^{MP} \\
\Delta y_t &= -\alpha_t r_t + \sigma_t^{D} \varepsilon_t^{D}
\end{aligned}
$$

Then, the previous model becomes

$$
\underbrace{[r_t,\ \Delta y_t]}_{y_t'} \underbrace{\begin{pmatrix} \frac{1}{\sigma_t^{MP}} & \frac{\alpha_t}{\sigma_t^{D}} \\ -\frac{\psi_t}{\sigma_t^{MP}} & \frac{1}{\sigma_t^{D}} \end{pmatrix}}_{A_t} = \underbrace{[\varepsilon_t^{MP},\ \varepsilon_t^{D}]}_{\varepsilon_t'}
$$

A class of models for structural analysis

A time-varying "structural" VAR model,

$$y_t' A_t = x_t' F_t + \varepsilon_t', \quad \varepsilon_t \sim N(0, I)$$

where

- A_t and F_t are "structural" parameters
- ε_t is a vector of "structural" shocks

Now, we face a similar challenge. How to specify prior over the time-varying structural parameters, A_t and F_t for $t = 1, ..., T$?

Observational equivalence

Following Rothenberg (1971), we say $(A_t, F_t)_{t=1}^T$ and $(\widetilde{A}_t, \widetilde{F}_t)_{t=1}^T$ are **observationally equivalent** if the likelihoods are equal for any $(\mathbf{y}_t)_{t=1}^T \in \mathbb{R}^{nT}$.

A similar (but not identical) proposition can be found in Bognanni (2018)

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Proposition 1

The time-varying structural parameters $(A_t, F_t)_{t=1}^T$ *and* $(\widetilde{A}_t, \widetilde{F}_t)_{t=1}^T$ *are observationally equivalent if and only if there exists orthogonal matrices* $(Q_t)_{t=1}^T \in \mathcal{O}_n^T$ *such that*

$$(A_t, F_t)_{t=1}^T = (\widetilde{A}_t Q_t, \widetilde{F}_t Q_t)_{t=1}^T$$

A similar (but not identical) proposition can be found in Bognanni (2018)

Rotation invariant condition and rotationally invariant prior

We say prior distribution for time-varying structural parameters, $p_S((A_t, F_t)_{t=1}^T)$, satisfy ***rotation invariant condition*** if

$$p_S((A_t, F_t)_{t=1}^T) = p_S((A_t Q_t, F_t Q_t)_{t=1}^T),$$

for every sequence of orthogonal matrices $(Q_t)_{t=1}^T \in \mathcal{O}_n^T$.

- We want our prior to treat observationally equivalent sequences of the structural parameters equally.
- "*A Bayesian analysis of a nonidentified model is always possible if a proper prior on all the parameters is specified*" (Poirier, 1998).

How to construct such prior?

We first reparameterize our model.

- Proposition 1 implies that our SVAR can be written in terms of time-varying orthogonal reduced-form parameters $(B_t, \Sigma_t, Q_t)_{t=1}^T$:

$$y_t' A_t = x_t' F_t + \varepsilon_t' \iff y_t' = x_t' B_t + \varepsilon_t' Q_t' h(\Sigma_t)$$
$$\text{where } \Sigma_t = h(\Sigma_t)' h(\Sigma_t)$$

- There is a mapping from the structural to the orthogonal reduced-form parameters:

$$f_h((A_t, F_t)_{t=1}^T) = (B_t, \Sigma_t, Q_t)_{t=1}^T$$

  - $B_t = F_t A_t^{-1}$
  - $\Sigma_t = (A_t A_t')^{-1}$
  - $Q_t = h((A_t A_t')^{-1}) A_t$

Mapping between structural and orthogonal reduced-form parameters

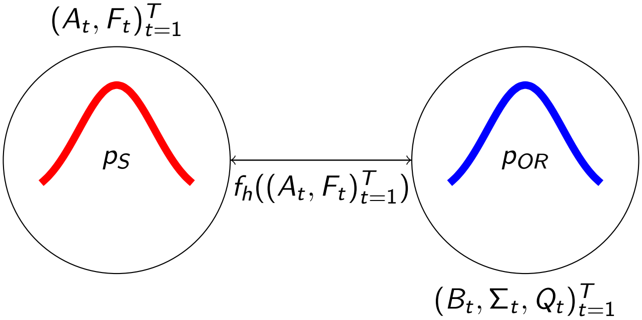
Structural param

$$(A_t, F_t)_{t=1}^T \xleftrightarrow[f_h((A_t, F_t)_{t=1}^T)]{} (B_t, \Sigma_t, Q_t)_{t=1}^T$$

Orthogonal Reduced-form param

Mapping between structural and orthogonal reduced-form parameters

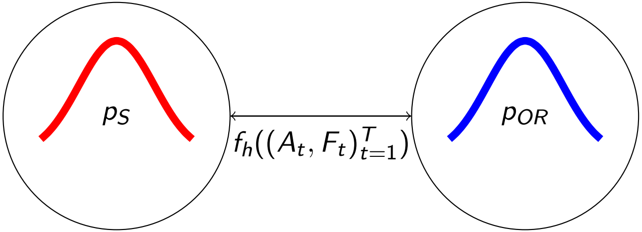
Let p_S be a prior density over the structural parameters induced by a prior density p_{OR} over the orthogonal reduced-form parameters $(B_t, \Sigma_t, Q_t)_{t=1}^T$



Mapping between structural and orthogonal reduced-form parameters

Let p_S be a prior density over the structural parameters induced by a prior density p_{OR} over the orthogonal reduced-form parameters $(B_t, \Sigma_t, Q_t)_{t=1}^T$

Goal: Rotational invariance

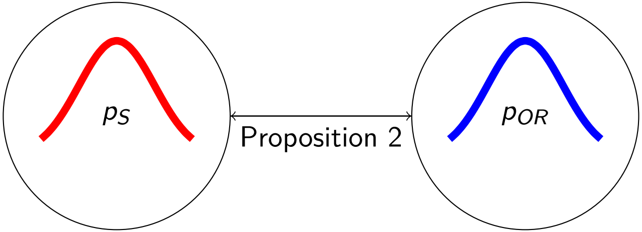


$$p((B_t, \Sigma_t, Q_t)_{t=1}^T) = \textbf{???}$$

Proposition 2

The prior over the time-varying structural parameters satisfies the rotation invariance condition if and only if the induced prior over the time-varying orthogonal reduced-form parameters must be independent over $(B_t, \Sigma_t)_{t=1}^T$ and $(Q_t)_{t=1}^T$, and the induced prior over $(Q_t)_{t=1}^T$ must be uniform with respect to the volume measure over $\mathcal{O}_n^T$

Rotational invariant $p((A_t, F_t)_{t=1}^T)$



$p((B_t, \Sigma_t)_{t=1}^T) \times \underbrace{p((Q_t)_{t=1}^T)}_{\text{Uniform}}$

Operationalization via Proposition 2

Begin with a reduced-form time-varying VAR

$$y_t' = x_t' B_t + e_t, \quad e_t \sim N(0, \Sigma_t)$$

Then, follow these steps:

1. Select prior on $B_{1:T}$ and $\Sigma_{1:T}$.
   - There are many alternatives for this as I introduced earlier. It does not have to be ordering invariant.
2. Select prior on $Q_{1:T}$.
   - Our proposition tells us that we should use uniform prior over $\mathcal{O}_n^T$.
3. Convert $p(B_{1:T}, \Sigma_{1:T})p(Q_{1:T})$ to $p_S(A_{1:T}, F_{1:T})$ using the mapping

$$(A_t, F_t)_{t=1}^T = f_h^{-1}\left((B_t, \Sigma_t, Q_t)_{t=1}^T\right)$$

Sign restrictions for sharper inference

For a given path, $(A_t^*, F_t^*)_{t=1}^{T}$, the set of $(A_t, F_t)_{t=1}^{T}$ that has the same likelihood can be very large.

- Economic theory sometimes helps reducing the set.

To sharpen inference, we impose **time-varying sign restrictions**.

Example of the sign restriction

Recall our simple two variable structural VAR,

$$\begin{aligned} r_t &= \psi \Delta y_t + \sigma^{MP} \varepsilon_t^{MP} \\ \Delta y_t &= -\alpha r_t + \sigma^{D} \varepsilon_t^{D} \end{aligned}$$

If we assume $\psi = 2$ and $\alpha = \sigma^{MP} = \sigma^{D} = 1$. Then, $\frac{\partial \Delta y_t}{\partial \varepsilon_t^{MP}} = -1/3$.

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However, observationally equivalent structural parameters would imply

$$\frac{\partial \Delta y_t}{\partial \varepsilon_t^{MP}} \in (-0.45, 0.45).$$

Example of the sign restriction

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However, observationally equivalent structural parameters would imply

$$\frac{\partial \Delta y_t}{\partial \varepsilon_t^{MP}} \in (-0.45, 0.45).$$

If we impose $\psi > 0$, then the set reduces to

$$\frac{\partial \Delta y_t}{\partial \varepsilon_t^{MP}} \in (-0.45, 0).$$

Time-varying sign restrictions

To sharpen inference, we impose **time-varying sign restrictions** of the following form

$$S_t(A_t, F_t) > 0 \quad \text{for } t = 1, ..., T.$$

- $S_t(A_t, F_t)$ is any continuous function whose range is $\mathbb{R}^{s_t}$ where s_t is the number of sign restrictions at time t.
- We only impose restrictions during the time that we know that they hold.

Example of time-varying sign restrictions

Recall our simple two variable time-varying structural VAR,

$$\begin{aligned} r_t &= \psi_t \Delta y_t + \sigma_t^{MP} \varepsilon_t^{MP} \\ \Delta y_t &= -\alpha_t r_t + \sigma_t^{D} \varepsilon_t^{D} \end{aligned}$$

Restriction:

- When the federal funds rate is the main policy instrument, we assume that

$$\psi_t > 0.$$

- In our application, we assume that the federal funds rate is the main policy instrument throughout our sample except for 1979Q4:1982Q4 (non-borrowed reserves targeting), 2009Q1:2015Q4 (ZLB), and 2020Q1:2021Q4 (COVID-19).

Rotation invariant prior with sign restrictions

Suppose $p_S((A_t, F_t)_{t=1}^T)$ is rotation invariant, then the following prior is rotation invariant as well,

$$p_{S*}((A_t, F_t)_{t=1}^T) \propto \underbrace{\quad p_S((A_t, F_t)_{t=1}^T) \quad}_{\text{rotationally invariant}} \times \underbrace{\prod_{t=1}^{T} 1\{S_t(A_t, F_t) > 0\}}_{\text{Sign restrictions}}.$$

- Sign restrictions truncate the domain of the original prior.
- For two observationally equivalent structural parameter sequences, $(A_t, F_t)_{t=1}^T$ and $(A_t Q_t, F_t Q_t)_{t=1}^T$ take the same density value under this prior as long as they satisfy sign restrictions.

Posterior inference

Posterior distribution can be written as

$$p((A_t, F_t)_{t=1}^{T} | (y_t)_{t=1}^{T}) \propto p((y_t)_{t=1}^{T} | (A_t, F_t)_{t=1}^{T}) p_S((A_t, F_t)_{t=1}^{T}) \prod_{t=1}^{T} 1\{S_t(A_t, F_t) > 0\}$$

We develop an MCMC algorithm that approximates the posterior distribution based on Metropolis-Hastings within Gibbs.

Main computational difficulty comes from the fact that we need to ensure that the entire sequence satisfies the sign restrictions.

Our algorithm allows for hyperparameters.

Application

Interpreting the recent policy tightening cycle

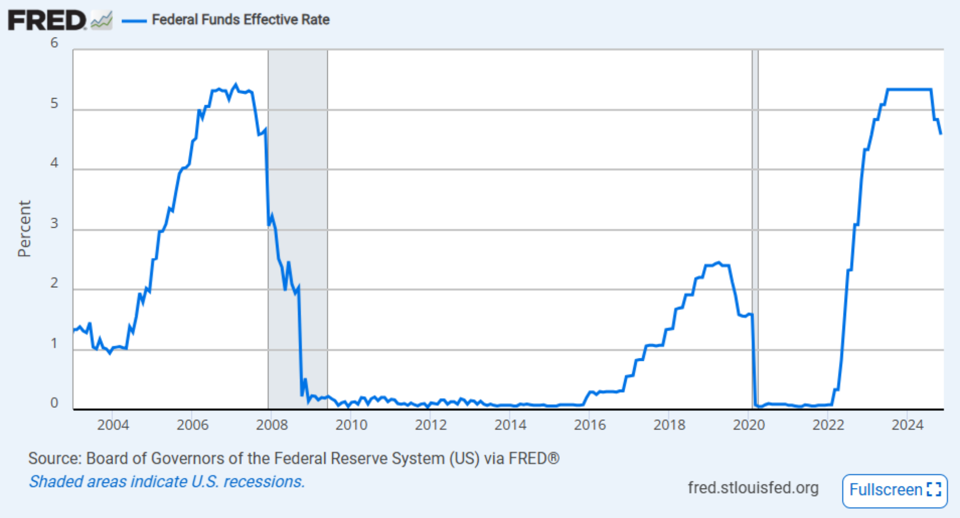
Interpreting the recent policy tightening cycle

Since the latest inflation run-up, most monetary policy discussions have revolved around the effects of interest rate increases on economic activity and inflation

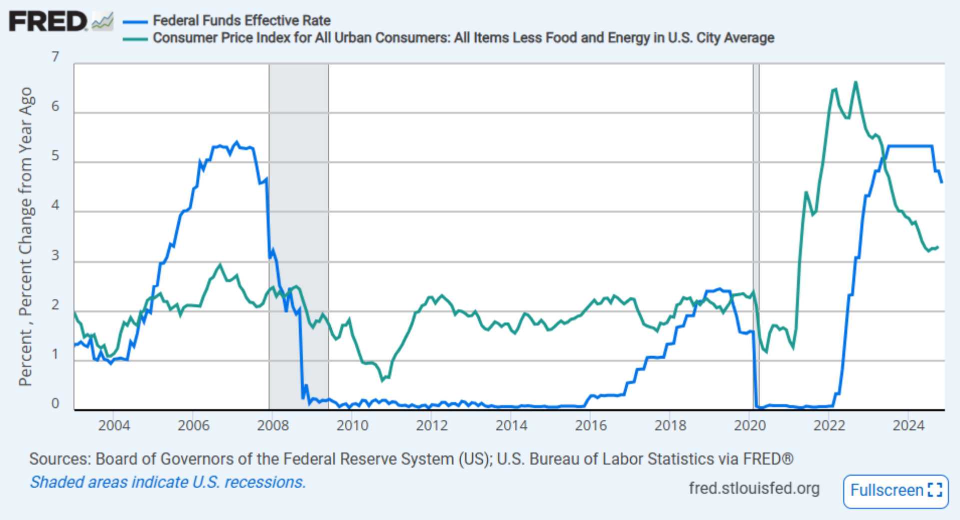
The Fed has a clear mandate to restore inflation to 2 percent, but there is uncertainty about how much interest rates have to increase to achieve the objective:

> *"Doing too little could allow above-target inflation to become entrenched and ultimately require monetary policy to wring more persistent inflation from the economy at a high cost to employment. Doing too much could also do unnecessary harm to the economy."* Powell (2023)

Interpreting the recent policy tightening cycle



Interpreting the recent policy tightening cycle



Interpreting the recent policy tightening cycle

We rely on our methodology to tackle:

1. How did the Federal Reserve respond to the state of the economy during the current policy tightening cycle?

2. How does the Fed's stance during this cycle compare with more Dovish or Hawkish monetary policy stances?

Data

We use a 5-variable quarterly model of the U.S. economy:

- Output growth (Δy_t, as measured by the log difference of real GDP)
- Inflation (π_t, as measured by the log difference of core PCE)
- The federal funds rate (r_t)
- Money growth (Δm_t, as measured by the log difference of M2)
- Moody's seasoned Baa corporate bond yield relative to the yield on 10-year treasury constant maturity

The sample runs from 1959:Q1 until 2023:Q4

An ordering invariant model with a rotation invariant prior

We estimate the TV-SVAR with the following prior

$$p((B_t, \Sigma_t, Q_t)_{t=1}^{T}) = \underbrace{p((B_t, \Sigma_t)_{t=1}^{T})}_{\text{RC-VAR}} \times \underbrace{p((Q_t)_{t=1}^{T})}_{\text{Uniform}}$$

Random correlations VAR (RC-VAR) with two lags

$$y_t = B_t x_{t-1} + e_t, \quad e_t \sim N(0, \Sigma_t)$$

$$\begin{aligned} \Sigma_t &= D_t C_t D_t \\ \delta_t &= 2\log(diag(D_t)) \\ \gamma_t &= \mathcal{G}(C_t) \end{aligned}$$

$$\begin{aligned} vec(B_t) &= vec(B_{t-1}) + \nu_t, \quad \nu_t \sim N(0, V_B) \\ \delta_t &= \delta_{t-1} + \eta_t, \quad \eta_t \sim N(0, V_\delta) \\ \gamma_t &= \gamma_{t-1} + \zeta_t, \quad \zeta_t \sim N(0, V_\gamma) \end{aligned}$$

From RC-VAR to TV-SVAR

$$y_t' A_t = x_t' F_t + \varepsilon_t' \iff y_t' = x_t' B_t + \varepsilon_t' Q_t{}' h(\Sigma_t) \text{ for } 1 \le t \le T$$

Sign restrictions for sharper inference

Let us focus on the monetary policy equation:

$$r_t = \underbrace{\psi_{\Delta y,t}\Delta y_t + \psi_{\pi,t}\pi_t + \psi_{\Delta m,t}\Delta m_t + \psi_{cs,t}cs_t}_{\text{Policy Reaction Function /Systematic Component}} + \ldots + \underbrace{\sigma_t^{MP}\varepsilon_t^{MP}}_{\textit{Shock}}$$

(one of the rows from the $A_t' y_t = F_t' x_t + \varepsilon_t$)

Sign restrictions for sharper inference

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(one of the rows from the $A_t' y_t = F_t' x_t + \varepsilon_t$)

Restriction 1: Following a monetary policy shock (ε_t^{MP}), the contemporaneous impulse responses of the price level and the stock of money are negative, and the contemporaneous impulse response of the federal funds rate is positive.

Sign restrictions for sharper inference

Let us focus on the monetary policy equation:

$$r_t = \underbrace{\psi_{\Delta y,t}\Delta y_t + \psi_{\pi,t}\pi_t + \psi_{\Delta m,t}\Delta m_t + \psi_{cs,t} cs_t}_{\text{Policy Reaction Function /Systematic Component}} + \ldots + \underbrace{\sigma_t^{MP}\varepsilon_t^{MP}}_{\text{Shock}}$$

Restriction 2:

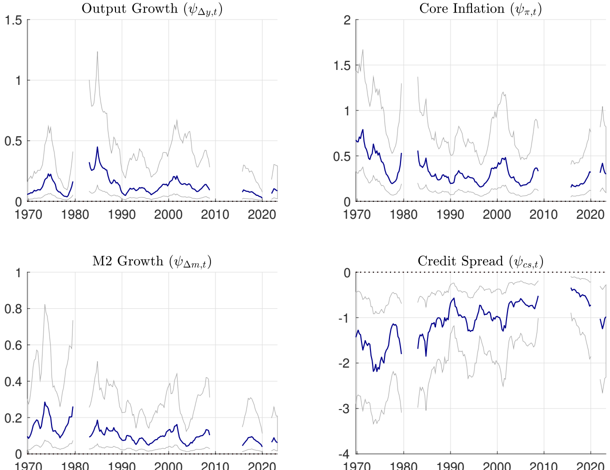
- When the federal funds rate is the main policy instrument we assume the following signs for coefficients in the policy equation:

$$\psi_{\Delta y,t} \in (0,4), \psi_{\pi,t} \in (0,4), \psi_{\Delta m,t} \in (0,4), \text{ and } \psi_{cs,t} \in (-4,0)$$

- We assume that the federal funds rate is the main policy instrument throughout our sample except for 1979Q4:1982Q4 (non-borrowed reserves targeting), 2009Q1:2015Q4 (ZLB), and 2020Q1:2021Q4 (COVID-19).

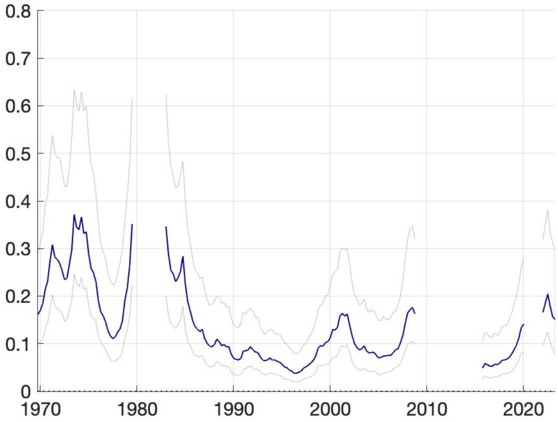
The systematic component of monetary policy

$$r_t = \underbrace{\psi_{\Delta y,t}\Delta y_t + \psi_{\pi,t}\pi_t + \psi_{\Delta m,t}\Delta m_t + \psi_{cs,t}cs_t}_{\text{Policy Reaction Function /Systematic Component}} + \ldots + \underbrace{\sigma_t^{MP}\varepsilon_t^{MP}}_{\textit{Shock}}$$



Standard deviation of the monetary policy shock

$$r_t = \underbrace{\psi_{\Delta y,t}\Delta y_t + \psi_{\pi,t}\pi_t + \psi_{\Delta m,t}\Delta m_t + \psi_{cs,t} cs_t}_{\text{Policy Reaction Function /Systematic Component}} + \ldots + \underbrace{\sigma_t^{MP}\varepsilon_t^{MP}}_{\text{Shock}}$$



The current monetary policy tightening cycle

Q: How did the Federal Reserve respond to the state of the economy during the current policy tightening cycle?

On March 16, 2022, the Fed raised the federal funds rate target, initializing the current monetary policy tightening cycle

Conditional on the date of lift-off, we investigate how the Federal Reserve responded to the state of the economy during 2022Q2:2023Q2

Answering this question will help us understand how much of the unexpected changes in the federal funds rate during this period came from the Fed's usual policy rules (responding to external shocks) versus unplanned policy shocks (deviation from the rule)

Historical decomposition – a simple example

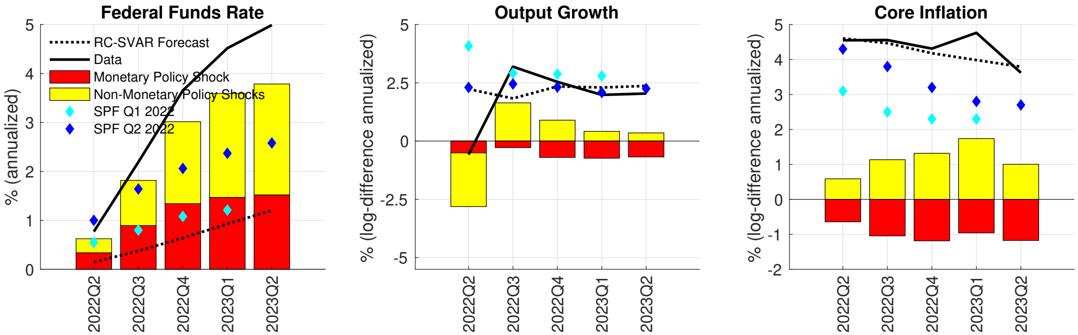
To fix ideas, consider a simplified constant parameters version of our model with two variables – real GDP growth (Δy_t) and the federal funds rate (r_t):

$$
\begin{aligned}
r_t &= \psi \Delta y_t + \ldots + \sigma_{MP} \varepsilon_t^{MP} \\
\Delta y_t &= -\alpha r_t + \ldots + \sigma_D \varepsilon_t^{D}
\end{aligned}
$$

It can be shown that

$$
r_t = \underbrace{\mathbf{E}_{t-1} r_t}_{\text{Predictable Component}} + \overbrace{\underbrace{\frac{\psi \sigma_D}{1+\alpha\psi}\varepsilon_t^{D}}_{\text{Systematic Response to } \varepsilon_t^{D}} + \underbrace{\frac{\sigma_{MP}}{1+\alpha\psi}\varepsilon_t^{MP}}_{\text{Monetary Policy Shock}}}^{\text{Unpredictable Component}}
$$

What did the Federal Reserve do?



Projections Made After the Release of 2022Q1 Data

Federal funds rate in 2023Q2 = 5.0%

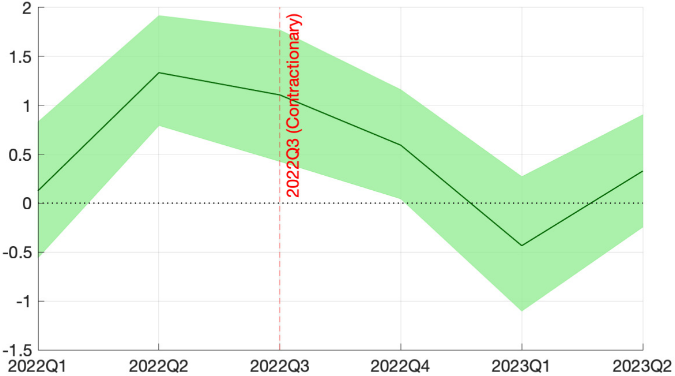
- Predictable (as of 2022Q1): 1.2%
- Monetary policy shocks: 1.5%
- Systematic policy reacting to other shocks: 2.3%

Monetary policy shock

Romer and Romer (2023) tentatively conclude that there was a monetary policy shock during the current tightening cycle

> *"It seems quite clear that a contractionary monetary shock occurred in the summer or early fall of 2022."*

Even though the definition of the shock in Romer and Romer (2023) is different than ours, the narrative approach and the SVAR approach are broadly in line.



Monetary Policy Shocks $\{\varepsilon_{r,t}\}_{t=2022Q1}^{2023Q2}$

$$r_t = \underbrace{\psi_{\Delta y,t}\Delta y_t + \psi_{\pi,t}\pi_t + \psi_{\Delta m,t}\Delta m_t + \psi_{cs,t}cs_t}_{\text{Policy Reaction Function /Systematic Component}} + \ldots + \underbrace{\sigma_t^{MP}\varepsilon_t^{MP}}_{Shock}$$

Hawkish vs. Dovish counterfactual simulations

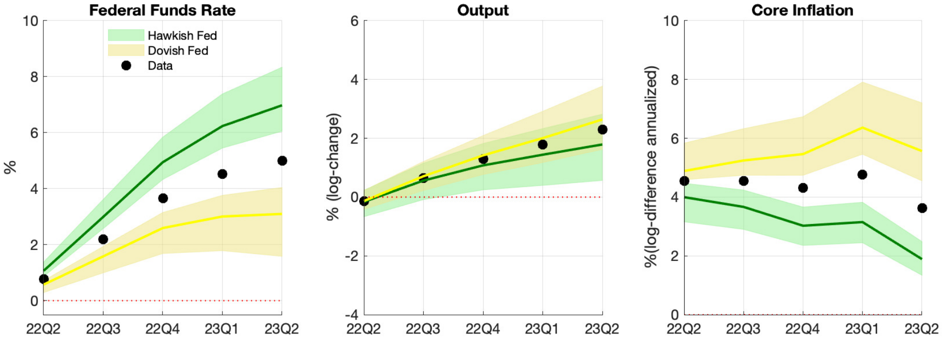
Q: How does the Fed's stance during this cycle compare with more Dovish or Hawkish monetary policy stances?

We conduct two counterfactual simulations where we replay history since 2022Q2, assuming that the Fed would have reacted to contemporaneous inflation differently than in our estimated policy rule:

$$r_t = \underbrace{\psi_{\Delta y,t}\Delta y_t + \psi_{\pi,t}\pi_t + \psi_{\Delta m,t}\Delta m_t + \psi_{cs,t}cs_t}_{\text{Policy Reaction Function /Systematic Component}} + \ldots + \underbrace{\sigma_t^{MP}\varepsilon_t^{MP}}_{\textit{Shock}}$$

- **"Hawkish Fed"**: we replace the model's estimated reaction to contemporaneous inflation with a reaction that is twice as large
- **"Dovish Fed"**: we replace the model's estimated reaction to contemporaneous inflation with a reaction that is half as large

Hawkish vs. Dovish counterfactual simulations



- In the Hawkish counterfactual, the output in the second quarter of 2023 would have been about 0.5 percent lower
- Under the Dovish counterfactual, the economy would have marginally overheated with output, and inflation would have run persistently above 5 percent

Summary

Reduced-form time-varying VARs,

$$y_t' = x_t' B_t + e_t', \quad e_t \sim N(0, \Sigma_t)$$

- **Ordering invariant prior** on $(B_t, \Sigma_t)_{t=1}^T$.
- Random-correlations VAR.

Structural time-varying VARs,

$$y_t' A_t = x_t' F_t + \varepsilon_t', \quad \varepsilon_t \sim N(0, I)$$

- **Rotation invariant prior** on $(A_t, F_t)_{t=1}^T$.
- Time-varying sign restrictions for sharper inference.

Possible extensions

Zero restrictions

Exploring the role of inference about the hyperparameters

Model selection via marginal likelihood comparisons

References

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